Discrete Choice Term Structure Models: Theory And Applications

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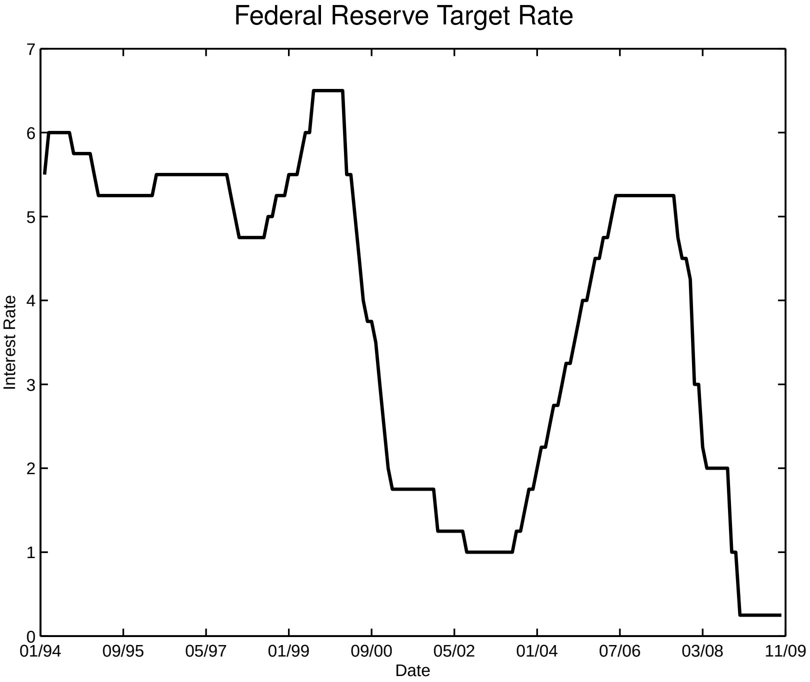

Federal Reserve Target Rate
Interest Rate
Date
0
1
2
3
4
5
6
7
01/94
09/95
05/97
01/99
09/00
05/02
01/04
07/06
03/08
11/09

Describing The Policy Function

- Like most other Central Banks, Fed changes its Target rate infrequently and chooses among multiples of 0.25%,
- Almost all of the policy changes followed a scheduled meeting of the Federal Open Market Committee [FOMC],
- Thus, modifications of the target rate occur at known dates while sizes of these modifications remain unknown until announced by the FOMC.
- Target rate have a discrete bounded support. This matters for modeling short term interest rate and derivative (Piazzesi (2005)).

Modeling the policy rule: the literature

- There is a huge literature on modeling the policy rule. Basically, given a level of the rate of inflation and economic activity, the FOMC decide a funds rate with the aim to close the gap between future rate of inflation and economic activity and their targets.
- Taylor(1979) introduce the following simple function,

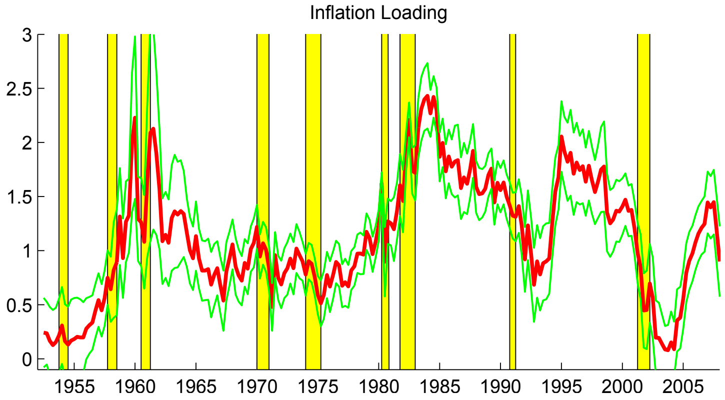
$$r_{t+1} = \beta_{r,\pi} E_t \pi_{t+1} + \beta_{r,x} x_{t+1},$$

  where $\beta_{r,\pi} > 1$ and $\beta_{r,x} < 1$. π_{t+1} is the inflation and x_{t+1} is the level of economic activity.
- Assuming that both inflation and economic activity distributions are absolutely continuous, this classic benchmark obviously does not reflect the discreteness nature of the fed fund rate.
- The target rate forecast can be negative, Black (1996) and Francisco Ruge-Murcia (2006) proposed to use Tobit model, but no closed form term structure of interest rate.

Modeling the policy rule: the literature

- Beside, recent evidences (Banzal and zhou (2002), Bikbov and and Chernov (2008), Cogley, T. and T. Sargent (2005), Sims, C. A. and T. Zha (2006), among others) suggest that both $\beta_{r,\pi}$ and $\beta_{r,x}$ are state dependent.
- This implies the introduction of monetary policy regimes. This approach have two mains drawbacks, which are the difficulties in the inference (estimation), and most importantly it does not implies neither discrete target rate, nor closed-form term structure of interest rate.
- Beside, in the regime switching approach the target rate can be negative. It is often difficult to interpret and to predict regimes.
- Hamilton and Jorda (JPE 2002) used the basic idea behind ordered response models to represent policy rule. They did not consider asset pricing implications.

Taylor Rule Coefficient Are Time-Varying: Inflation coefficient from Ang et al. (2009)



Modeling the policy rule: our approach

- We follow Hamilton and Jorda (JPE 2002) by using ordered response models to represent policy rule. In addition, we provide a general framework that allows to compute the term structure of interest rate as well as short rate's derivatives in closed-form.
- The non-linearity of the model allows us to generate state dependent parameters $\beta_{r,\pi}$ and $\beta_{r,x}$: modeling the discreteness of the target rate naturally implies asymmetries and state dependent policy rule.
- To our knowledge the only other existing discrete dynamic model that allows closed-form asset-price is the Integer-Value Autoregressive [INAR] process (Gourieroux and Jasiak, 2002). But the INAR process can allow only positive and positively correlated state variables.

Outline

- Our model for the policy rule.
- Pricing kernel specification and asset pricing implications.
- Some empirical results.

Representing The Policy Function : the level of the target rate

- A simple representation of these empirical features is given by,

$$r_{t+1} = (1 - I_{t+1})r_t + I_{t+1}c\eta_{t+1} \tag{1}$$

  where I_{t+1} is a deterministic indicator function equal to 1 when a meeting of the FOMC is scheduled at $t+1$ and 0 otherwise.

- $c = 0.0025$ and η_{t+1} is an integer-valued stochastic process with support given by,

$$\eta_{t+1} \in \{\underline{n}, ..., \bar{n}\}.$$

- The Target rate level ranges from $c\underline{n}$ to $c\bar{n}$.
- In particular, setting $\underline{n} = 0$ guarantees that r_t remains non-negative.

Representing The Policy Function : Change in the target rate

- In practice, for the level of the target rate, we have $\underline{n} = 0$ and $\bar{n} = 25$, which is practically very difficult to handle. That is why examining also model for the change in the target rate can be beneficial.
- Especially because the target rate change usually takes few values, in general $0.0025 \times \{-2, -1, 0, 1, 2\}$.
- A simple representation of the discreteness in the target rate change is given by,

$$r_{t+1} - r_t = I_{t+1} c \eta_{t+1} \tag{2}$$

where I_{t+1} is a deterministic indicator function equal to 1 when a meeting of the FOMC is scheduled at $t+1$ and 0 otherwise.

Representing The Policy Function : Change in the target rate

- $c = 0.0025$ and η_{t+1} is an integer-valued stochastic process with support given by,

$$\eta_{t+1} \in \{\underline{n}, ..., \bar{n}\} .$$

- The Target rate change ranges from $c\underline{n}$ to $c\bar{n}$.
- Contrary to the level, we cannot guaranty that the target rate is always positive.
- We consider the two approaches in the paper (Level and change).

Dynamic process with discrete and bounded support: the general approach

Note that all the stochastic properties of the Target rate are derived from those of η_{t+1}.

- The classic approach in micro-econometric literature (probit, logit or ordered response models, see Hausman, Lo, and MacKinlay (1992)) is to ask what is the decision-making process behind policy decision, and to assume that conditional on available information at time t, and given the observation of the state of the economy Y_{t+1}, the FOMC set its Target according to the following rule.

$$\eta_{t+1} = n \Longleftrightarrow \delta_{n-1,t}\left(Y_{t+1}\right) \leq \tilde{\eta}_{t+1} \leq \delta_{n,t}\left(Y_{t+1}\right),$$

where $\tilde{\eta}_{t+1}$ summarizes the state of the economy, as perceived by members of the FOMC. This is unobserved to the econometrician.

Dynamic process with discrete and bounded support: the general approach

- $\delta_{n,t}(Y_{t+1})$ is an increasing sequence, which might depend on the state vector Y_{t+1}.
- Let denote the Conditional cumulative distribution of $\tilde{\eta}_{t+1}$ by CDF_t
- we thus have

$$Proba_t[\eta_{t+1} = n | Y_{t+1}] = CDF_t(\delta_{n,t}(Y_{t+1})) - CDF_t(\delta_{n-1,t}(Y_{t+1})),$$

for $\underline{n} < n \leq \bar{n}$

- Models will differ only by the choice of $CDF_t(\delta_{n,t}(Y_{t+1}))$, or equivalently the choice of the cumulative distribution function of $\tilde{\Delta}_{t+1}$ and the increasing sequence of thresholds $\delta_{n,t}$.

Dynamic process with discrete and bounded support: the general approach

- Also we have

$$Proba_t[\eta_{t+1} \leq n | Y_{t+1}] = CDF_t(\delta_{n,t}(Y_{t+1})),$$

- Hamilton and Jorda (JPE 2002) chose $CDF_t(x) = \Phi\left(\frac{x-\beta' Y_{t+1}}{\sigma}\right)$ and $\delta_{n,t} = \delta_n$ an increasing and time invariant sequence, $\Phi(\cdot)$ is the cdf of a standard normal distribution.

Our choice of $Proba_t[\eta_{t+1} \leq n | Y_{t+1}]$

- The Hamilton and Jorda (JPE 2002) set-up cannot allows closed form term structure of interest rate.
- One of the goal of this paper is to link dynamic ordered response models for the target rate to the term structure of interest rate.
- Obviously we need to choose another type of c.d.f.
- One of the key ingredient in asset pricing is the conditional moment generating function.

Our choice of $Proba_t[\eta_{t+1} \leq n | Y_{t+1}]$

- The conditional moment generating function of η_{t+1}, is

$$
\begin{aligned}
E_t[\exp(u\eta_{t+1}) | Y_{t+1}] &= \sum_{n=\underline{n}}^{\bar{n}} \exp(un) Proba_t[\eta_{t+1} = n | Y_{t+1}] \\
&= \sum_{n=\underline{n}}^{\bar{n}} \exp(un) \left\{ CDF_t(\delta_{n,t}) - CDF_t(\delta_{n-1,t}) \right\},
\end{aligned}
$$

- Because our goal is to have closed form solution, we depart from the normality, and choose the exponential distribution

$$
CDF_t(x) = 1 - \exp(-x\lambda_{t+1})
$$

$$
Proba_t[\eta_{t+1} \leq n | Y_{t+1}] = CDF_t(\delta_{n,t}(Y_{t+1})) = 1 - \exp(-\delta_{n,t+1}\lambda_{t+1})
$$

Our choice of $\delta_{n,t+1}$ and λ_{t+1}

- The issue now is to choose a sequence of increasing and positive $\delta_{n,t+1}$, and positive variable λ_{t+1}.
- We can adopt The Hamilton and Jorda (JPE 2002) set up by fixing $\delta_{n,t+1} = \delta_n$, time invariant. In order to have closed-form term structure of interest rate we can also fix

$$\lambda(Y_{t+1}) = a + b' Y_{t+1} + Y'_{t+1} \gamma Y_{t+1} > 0$$

- We can impose the positivity of $\lambda(Y_{t+1})$ by restricting γ to be definite positive and $a > \frac{1}{4} b' \gamma^{-1} b$: We have considered this approach to model changes in the target rate.
- We could have choose $\lambda(Y_{t+1}) = a + b' Y_{t+1}$, but in addition to $a > 0$ and $b \geq 0$ this constraints us to positive state variables.

Our choice of $\delta_{n,t+1}$ and λ_{t+1}

- In general we cannot identify both $\delta_{n,t+1}$ and λ_{t+1} separately, we provide a much more general result.
- $\delta_{n,t+1}\lambda_{t+1} = \alpha_n + \beta_n' Y_{t+1} + Y_{t+1}' \Gamma_n Y_{t+1}$.
- Because $\delta_{n,t+1}\lambda_{t+1}$ is an increasing and positive sequence we impose the following restrictions :
  1. $\Gamma_n - \Gamma_{n-1}$ is positive definite,
  2. $\alpha_n - \alpha_{n-1} \geq \frac{1}{4}(\beta_n - \beta_{n-1})'(\Gamma_n - \Gamma_{n-1})^{-1}(\beta_n - \beta_{n-1})$

Example of sequences

One simple way to impose
$\alpha_n - \alpha_{n-1} + (\beta_n - \beta_n)' Y_{t+1} + Y'_{t+1} \left(\Gamma_n - \Gamma_{n-1} \right) Y_{t+1} \geq 0$ is just to set
$\alpha_n - \alpha_{n-1} + (\beta_n - \beta_n)' Y_{t+1} + Y'_{t+1} \left(\Gamma_n - \Gamma_{n-1} \right) Y_{t+1} =$
$(Y_{t+1} - \rho_n)' \Omega_n (Y_{t+1} - \rho_n)$ where Ω_n is positive definite. This implies

$$
\begin{aligned}
\Gamma_n - \Gamma_{n-1} &= \Omega_n \\
\beta_n - \beta_{n-1} &= -2\Omega_n \rho_n \\
\alpha_n - \alpha_{n-1} &= \rho'_n \Omega_n \rho_n
\end{aligned}
$$

we adopt this strategy to model the level of the target rate.
Flexibility in the choices of sequences impart tremendous flexibility to fit the dynamic properties of the Target rate.

Dynamic of Y_{t+1} and Joint Conditional Moment Generating Function

We assume that Y_{t+1} follows a $VAR(1)$, i.e

$$Y_{t+1} = \omega + \phi Y_t + \epsilon_{t+1},$$

where $\epsilon_{t+1} \sim iidN\left(0, \Sigma\Sigma'\right)$.
The conditional moment generating function, $\mathcal{M}_X(u)$, of $X_{t+1} = (r_{t+1}, Y'_{t+1})'$ is defined as

$$\mathcal{M}_X(u) \equiv E_t\left[\exp\{u^T X_{t+1}\}\right],$$

and, in the case of DCDTSM, is given by

$$\mathcal{M}_X(u) = \sum_{n=\underline{n}}^{\bar{n}} \exp\left(A_n(u) + B_n\left(u\right)^T Y_t + Y_t^T C_n\left(u\right) Y_t\right),$$

where the coefficients $A_n(u)$, $B_n(u)$ and $C_n(u)$ depends on the parameters of the models.

Asset Pricing

- We define the pricing kernel directly,

$$\xi_{t+1} = \xi^{Y}_{t+1}\xi^{r}_{t+1},$$

- $$\xi^{Y}_{t+1} = \frac{\exp\left(\lambda'_{y,t}Y_{t+1}\right)}{E_t\left[\exp\left(\lambda'_{y,t}Y_{t+1}\right)\right]}$$

- $$\xi^{r}_{t+1} = \frac{Q_{t|Y_{t+1}}\left[r_{t+1}/c\right]}{P_{t|Y_{t+1}}\left[r_{t+1}/c\right]},$$

  for a choice of the risk-neutral probability, $Q_{t|Y_{t+1}}$. We assume that the risk-neutral is given by,

$$Q_{t|Y_{t+1}}[r_{t+1} \leq nc] = \exp\left(\alpha^*_n + \beta^{*\prime}_n Y_{t+1} + Y'_{t+1}\Gamma^*_n Y_{t+1}\right),$$

  so that the joint distribution belongs to the same family under each measure.

Equivalence between P and Q

- No-Arbitrage opportunities does not exist if the two probability measures are equivalent, i.e they must have the same set of impossible events.
- At each date t, we must have that

$$P_{t|Y_{t+1}}[r_{t+1} = nc] = 0 \Longleftrightarrow Q_{t|Y_{t+1}}[r_{t+1} = nc] = 0$$

- In our set-up, we show that imposing No-Arbitrage opportunities is equivalent to:

$$\alpha_n - \alpha_{n-1} = 0.25\left(\beta_n - \beta_{n-1}\right)'\left(\Gamma_n - \Gamma_{n-1}\right)^{-1}\left(\beta_n - \beta_{n-1}\right).$$

$$\Leftrightarrow$$

$$\beta_n^* - \beta_{n-1}^* = \left(\Gamma_n^* - \Gamma_{n-1}^*\right)\left(\Gamma_n - \Gamma_{n-1}\right)^{-1}\left(\beta_n - \beta_{n-1}\right)$$

and

$$\alpha_n^* - \alpha_{n-1}^* = 0.25\left(\beta_n^* - \beta_{n-1}^*\right)'\left(\Gamma_n^* - \Gamma_{n-1}^*\right)^{-1}\left(\beta_n^* - \beta_{n-1}^*\right).$$

Equivalence between P and Q

In the following parametrization

$$
\begin{aligned}
\Gamma_n - \Gamma_{n-1} &= \Omega_n \\
\beta_n - \beta_{n-1} &= -2\Omega_n \rho_n \\
\alpha_n - \alpha_{n-1} &= \rho_n' \Omega_n \rho_n
\end{aligned}
$$

The equivalence condition between P and Q implies that $\rho_n = \rho_n^*$

Asset Pricing

- we use the standard affine specification for $\lambda_{y,t}$,

$$\lambda_{y,t} = \lambda_0 + \lambda_1 Y_t,$$

which implies that

$$Y_{t+1} = \omega^* + \phi^* Y_t + \epsilon^*_{t+1},$$

where

$$\begin{aligned} \omega^* &= \omega + \Sigma\Sigma' \lambda_0 \\ \phi^* &= \phi + \Sigma\Sigma' \lambda_1 \\ \epsilon^*_{t+1} &\sim {}^{Q} iidN\left(0, \Sigma\Sigma'\right). \end{aligned}$$

Bond and Option Prices

- The price of a zero coupon bond with maturity $m = h + 1$, $D_t(m)$, is given by

$$
\begin{aligned}
e^{r_t} D_t(m) &= E_t^Q \left[\exp\left(-\sum_{j=0}^{h} r_{t+j} \right) \right] \\
&= \sum_{(n_1,\ldots,n_h)} \exp\left(a_{(n_1,\ldots,n_h)} + b_{(n_1,\ldots,n_h)}^T Y_t + Y_t^T c_{(n_1,\ldots,n_h)} Y_t \right),
\end{aligned}
$$

- the prices of a call option on Fed funds rate at maturity $m = h + 1$ and strike rate k, $C_t(m, k)$ is given by

$$
\begin{aligned}
\frac{e^{r_t} C_t(m,k)}{c} &= E_t^Q \left[\exp\left(-\sum_{j=0}^{h} r_{t+j} \right) \left(r_{t+h+1} - k \right)^+ \right] \\
&= \sum_{(n_1,\ldots,n_h)} \sum_{j=k/c}^{\bar{n}} \left[\begin{array}{c} \exp\left(a_{(n_1,\ldots,n_h)} + b_{(n_1,\ldots,n_h)}^T Y_t + Y_t^T c_{(n_1,\ldots,n_h)} Y_t \right) \\ - \exp\left(a_{(j,n_1,\ldots,n_h)} + b_{(j,n_1,\ldots,n_h)}^T Y_t + Y_t^T c_{(j,n_1,\ldots,n_h)} Y_t \right) \end{array} \right]
\end{aligned}
$$

Bond and Option Prices

- The sum is taken on the set h-tuples. The coefficients are given by the following recursions,

$$
\begin{aligned}
a_{(n_1,\ldots,n_h)} = & A^*\left(b_{(n_1,\ldots,n_{h-1})} + \beta^*_{n_h}, c_{(n_1,\ldots,n_{h-1})} + \Gamma^*_{n_h}\right) \\
& + a_{(n_1,\ldots,n_{h-1})} + \ln\left(1 - \exp\left(-c\right)\right)^{1_{[n_h<\bar{n}]}} - c n_h + \alpha^*_{n_h} \\
b_{(n_1,\ldots,n_h)} = & B^*\left(b_{(n_1,\ldots,n_{h-1})} + \beta^*_{n_h}, c_{(n_1,\ldots,n_{h-1})} + \Gamma^*_{n_h}\right) \\
c_{(n_1,\ldots,n_h)} = & C^*\left(c_{(n_1,\ldots,n_{h-1})} + \Gamma^*_{n_h}\right).
\end{aligned}
$$

and with initial conditions given by
$a_n = \ln\left(1 - \exp\left(-c\right)\right)^{1_{[n<\bar{n}]}} - cn + \alpha^*_n + A\left(\beta^*_n, \Gamma^*_n\right)$,
$b_n = B\left(\beta^*_n, \Gamma^*_n\right)$ and $c_n = C\left(\Gamma^*_n\right)$.

Data

	Central moments				Autocorrelations		
Mat	Mean	Stdev	Skew	Kurt	Lag1	Lag2	Lag3
3	3.726	1.894	-0.460	1.685	0.979	0.952	0.920
4	3.742	1.907	-0.448	1.673	0.978	0.951	0.920
5	3.758	1.916	-0.436	1.670	0.978	0.950	0.919
6	3.775	1.920	-0.426	1.672	0.977	0.950	0.918
7	3.793	1.922	-0.417	1.678	0.976	0.949	0.917
8	3.811	1.921	-0.409	1.686	0.976	0.948	0.916
9	3.829	1.919	-0.402	1.696	0.975	0.946	0.915
10	3.847	1.914	-0.396	1.707	0.974	0.945	0.913
11	3.865	1.908	-0.391	1.719	0.974	0.944	0.912
12	3.883	1.900	-0.386	1.731	0.973	0.943	0.911
r_t	3.816	1.968	-0.475	1.683	0.982	0.959	0.929
π_t	2.250	0.423	-0.494	3.107	0.949	0.896	0.834
EG	0.984	1.754	-1.309	4.237	0.971	0.932	0.887

Policy rule estimation

- For a given sample of target rate data $(r_1, r_2, ..., r_T)$,
- Compute $\eta_t = r_t/c$, η_t is an integer whose support is $(\underline{n}, \bar{n})$
- The likelihood is

$$L(r_1, r_2, ..., r_T) = \prod_{t=1}^{T} Proba_{t-1}[\eta_t = r_t/c | Y_t]$$

- with

$$Proba_{t-1}[\eta_t = r_t/c | Y_t] = \exp\left(\alpha_{r_t/c} + \beta'_{r_t/c} Y_t + Y_t' \Gamma_{r_t/c} Y_t\right) - \exp\left(\alpha_{-1+r_t/c} + \beta'_{-1+r_t/c} Y_t + Y_t' \Gamma_{-1+r_t/c} Y_t\right)$$

Parametric Specification

$$Proba_{t-1}[r_t/c \leq n | Y_t] = \exp\left(\alpha_n + \beta_n' Y_t + Y_t' \Gamma_n Y_t\right)$$
$$Proba_{t-1}[(r_t - r_{t-1})/c \leq n | Y_t] = \exp\left(\alpha_n + \beta_n' Y_t + Y_t' \Gamma_n Y_t\right)$$

- For the level,

$$\begin{aligned} \Gamma_n - \Gamma_{n-1} &= \Omega_n,\ \beta_n - \beta_{n-1} = -2\Omega_n \rho_n \\ \alpha_n - \alpha_{n-1} &= \rho_n' \Omega_n \rho_n \\ \rho_n &= n^{\kappa_1} \rho,\ \Omega_n = n^{\kappa_2} \Omega \end{aligned}$$

- For change,

$$\alpha_n = c_n \alpha, \quad \beta_n = c_n \beta, \quad \Gamma_n = c_n \Gamma$$

with $c_{\bar{n}} = 0$, $c_0 = -1$ and

$$c_n = c_0 + \frac{c_{\bar{n}} - c_0}{1 + \exp(c_{up})} \forall 0 \leq n \leq \bar{n}$$
$$c_n = c_{\underline{n}} + \frac{c_0 - c_{\underline{n}}}{1 + \exp(c_{down})} \ \forall \underline{n} \leq n \leq 0.$$

Macroeconomic Dynamics : VAR(1) estimation on Y_t

ω_π	0.1292	ω_x	0.3190				
	(0.0535)		(0.1050)				
ϕ_π	0.9328	$\phi_{x,\pi}$	-0.1710	$\phi_{\pi,x}$	0.0151	ϕ_x	1.0270
	(0.0244)		(0.0479)		(0.0059)		(0.0117)
σ_π^2	0.0152	$\sigma_{x,\pi}$	0.0108	σ_x^2	0.0589		
	(0.0016)		(0.0023)		(0.0062)		
ML		-321.2895		BIC		2.1328	

Estimation of Monetary Policy Rule: level

DCDTS

β_π	0.1748	β_x	0.1668				
	(0.9724)		(3.1753)				
Γ_π	2.8847	$\Gamma_{\pi,x}$	0.8796	Γ_x	0.2682		
	(1.6734)		(0.4882)		(0.1498)		
κ_1	0.5961	κ_2	-1.3406	RMSE	0.88		
	(0.1435)		(0.2108)				

Linear Benchmark

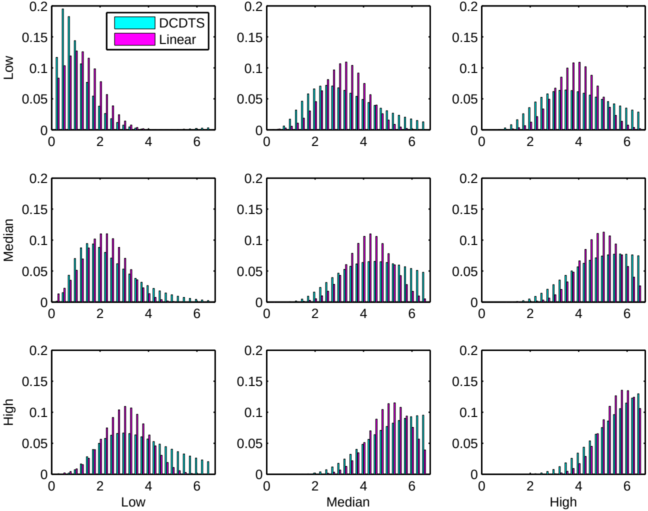
ω_r	-1.0504	σ_r^2	0.8292		
	(0.0535)		(0.0479)		
$\beta_{r,\pi}$	1.8502	$\beta_{r,x}$	0.7141	RMSE	0.91
	(0.1050)		(0.0244)		

Estimation of Monetary Policy Rule: Change

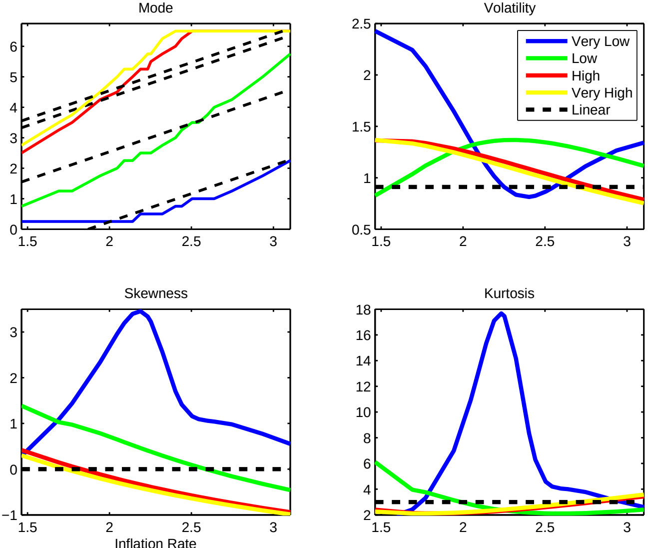
DCDTS					
α	0.4016				
	(0.1735)				
β_π	-0.2341	β_x	0.1371		
	(0.1299)		(0.0527)		
Γ_π	0.0390	$\Gamma_{\pi,x}$	-0.0245	Γ_x	0.0159
	(0.0256)		(0.0111)		(0.0054)
c_0	-1	$c_{\min}$	-29.913	$c_{\max}$	0
			(7.0191)		
c_{down}	-0.5328	c_{up}	-2.6395	RMSE	0.2040
	(0.1071)		(0.7254)		

Linear Benchmark					
ω_r	0.2217	σ_r^2	0.0458		
	(0.0925)		(0.0048)		
$\beta_{r,\pi}$	-0.1325	$\beta_{r,x}$	0.0478	RMSE	0.2141
	(0.0422)		(0.0101)		

Fed fund rate conditional distribution



Fed fund rate conditional moments



Taylor Rule

1. The most common representation of monetary policy is through the Taylor rule
2. In the Taylor rule, interest rate is linear in inflation and the output gap. Taylor-rule can be written as

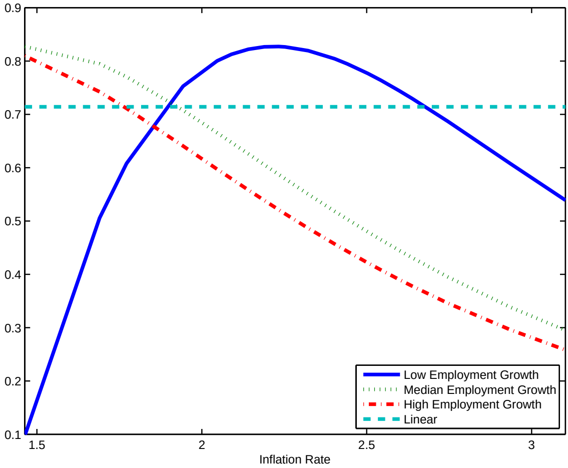
$$E_t\left[r_{t+1} | \pi_{t+1}, x_{t+1}\right] = a + a_\pi \pi_{t+1} + a_x x_{t+1}$$

3. For comparison purposes, our implied rule can be approximated as

$$E_t\left[r_{t+1} | Y_{t+1}\right] \approx$$
$$b\left(Y_t\right) + b_\pi\left(Y_t\right)\left(\pi_{t+1} - E_t[\pi_{t+1}]\right) + b_x\left(Y_t\right)\left(x_{t+1} - E_t[x_{t+1}]\right)$$

where the coefficients depend on the state of the economy and on model parameters.

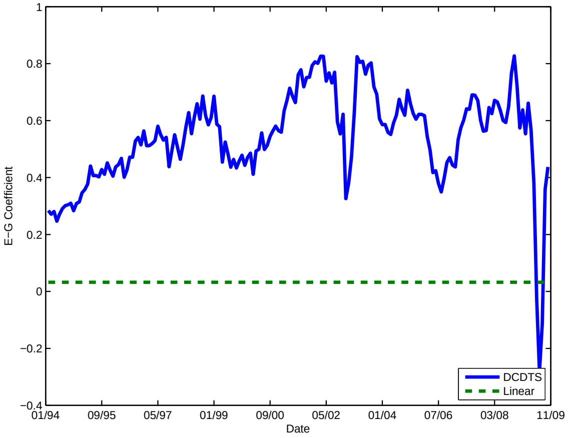
Employment Growth Coefficient



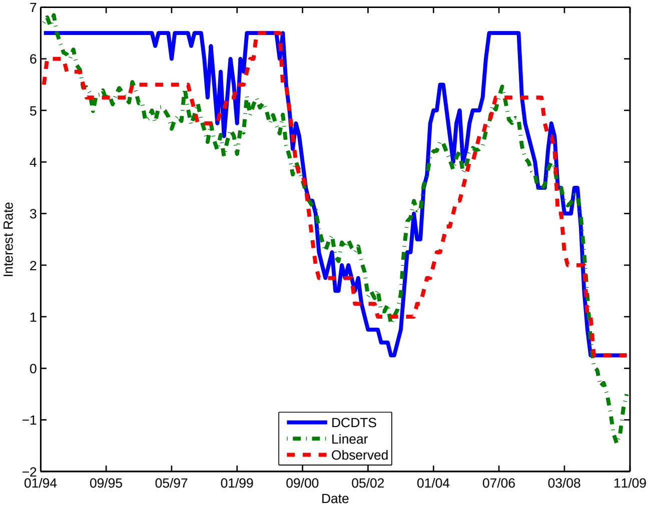
Employment Growth Coefficient : comments

- DCDTS and VAR coefficients are close for median inflation and employment growth rates.
- For a given employment growth rate, the response of the Fed to employment decreases with inflation. The long-term price stability mandate becomes more important as inflation becomes more pervasive.
- The response decreases to zero for low inflation and employment rates, as the Target rate approaches zero.
- For a given inflation rate, the response to employment increases with a deteriorating state. The short-term economic growth mandate becomes more important as we enter a recession.

Time series of Employment Growth Coefficient



Target Rates From Data And Models



In-Sample Forecasting of the target rate : the RMSE

	In sample				
h	DCDTS-C	Linear-C	RW	DCDTS-L	Linear-L
1	0.301	0.331	0.230	0.851	0.889
2	0.402	0.451	0.368	0.828	0.879
3	0.497	0.567	0.512	0.823	0.871
4	0.590	0.675	0.652	0.824	0.865
5	0.681	0.777	0.783	0.808	0.858
6	0.777	0.877	0.907	0.820	0.857
7	0.881	0.978	1.028	0.822	0.860
8	0.981	1.070	1.149	0.821	0.871
9	1.095	1.171	1.262	0.796	0.891
10	1.203	1.265	1.379	0.820	0.917
11	1.312	1.363	1.489	0.857	0.951
12	1.414	1.456	1.594	0.896	0.985

Out-of-Sample Forecasting of the target rate : the RMSE

	Out-of-Sample				
h	DCDTS-C	Linear-C	RW	DCDTS-L	Linear-L
1	0.361	0.384	0.257	1.881	1.200
2	0.496	0.538	0.400	1.936	1.208
3	0.629	0.692	0.559	1.903	1.214
4	0.755	0.837	0.714	1.849	1.210
5	0.871	0.968	0.858	1.783	1.200
6	0.998	1.096	0.991	1.707	1.186
7	1.131	1.213	1.123	1.615	1.172
8	1.259	1.310	1.255	1.456	1.152
9	1.406	1.415	1.380	1.304	1.141
10	1.543	1.504	1.515	1.112	1.146
11	1.681	1.620	1.644	0.908	1.173
12	1.813	1.737	1.770	0.971	1.210

In-Sample Conditional Forecasting of the target rate : the RMSE

	In sample				
h	DCDTS-C	Linear-C	RW	DCDTS-L	Linear-L
1	0.216	0.331	0.188	0.757	1.359
2	0.252	0.425	0.276	0.699	1.400
3	0.326	0.536	0.331	0.697	1.386
4	0.393	0.623	0.400	0.638	1.350
5	0.468	0.697	0.469	0.583	1.319
6	0.552	0.757	0.560	0.576	1.283
7	0.652	0.781	0.651	0.534	1.240
8	0.756	0.782	0.739	0.559	1.186
9	0.880	0.803	0.839	0.481	1.121
10	0.980	0.800	0.953	0.486	1.058
11	1.097	0.869	1.057	0.483	1.008
12	1.215	0.959	1.161	0.466	0.956

Out-of-Sample Conditional Forecasting of the target rate : the RMSE

	Out-of-Sample				
h	DCDTS-C	Linear-C	RW	DCDTS-L	Linear-L
1	0.204	0.326	0.242	0.788	1.414
2	0.249	0.460	0.285	0.737	1.476
3	0.324	0.607	0.348	0.755	1.499
4	0.394	0.739	0.406	0.708	1.498
5	0.475	0.856	0.477	0.650	1.472
6	0.563	0.950	0.573	0.640	1.425
7	0.658	0.996	0.641	0.591	1.372
8	0.750	0.985	0.729	0.602	1.278
9	0.857	0.954	0.829	0.494	1.153
10	0.939	0.860	0.921	0.480	1.045
11	1.015	0.846	1.012	0.461	0.962
12	1.127	0.883	1.114	0.443	0.908

Pricing Bond

To estimate the risk-neutral model, define the bond valuation errors as

$$\varepsilon_i = y_i^{Mkt} - y_i^{Mod},$$

where y_i^{Mkt} represents the yield associated to the market price of the i^{th} bond, y_i^{Mod} represents the yield associated to the model price. Assume these disturbances are i.i.d. normal so that the option log likelihood is

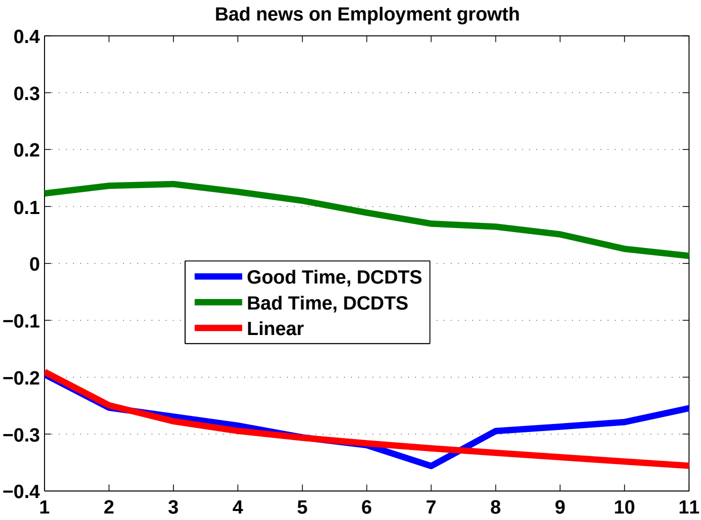
$$\ln L^B \propto -\frac{1}{2}\sum_{i=1}^{N}\left\{\ln\left(s_\varepsilon^2\right) + \varepsilon_i^2/s_\varepsilon^2\right\}. \tag{3}$$

where we can concentrate out s_ε^2 using the sample analogue $\hat{s}_\varepsilon^2 = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i^2$.

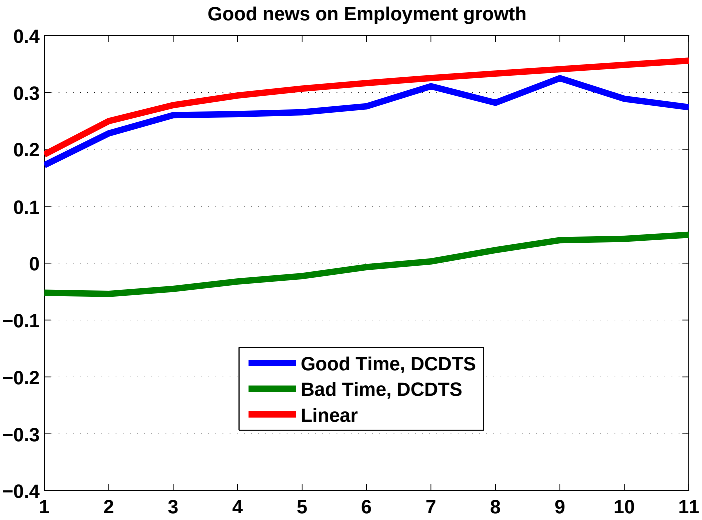
Pricing Bond

	In sample		Out-of-Sample	
Maturity	DCDTS-C	Linear-C	DCDTS-C	Linear-C
3	0.223	0.234	0.293	0.287
4	0.222	0.250	0.312	0.304
5	0.232	0.274	0.323	0.321
6	0.246	0.298	0.329	0.339
7	0.261	0.323	0.331	0.357
8	0.274	0.347	0.333	0.377
9	0.284	0.370	0.336	0.398
10	0.295	0.392	0.342	0.420
11	0.309	0.413	0.350	0.442
12	0.325	0.434	0.333	0.465
Total	**0.269**	**0.340**	**0.328**	**0.376**

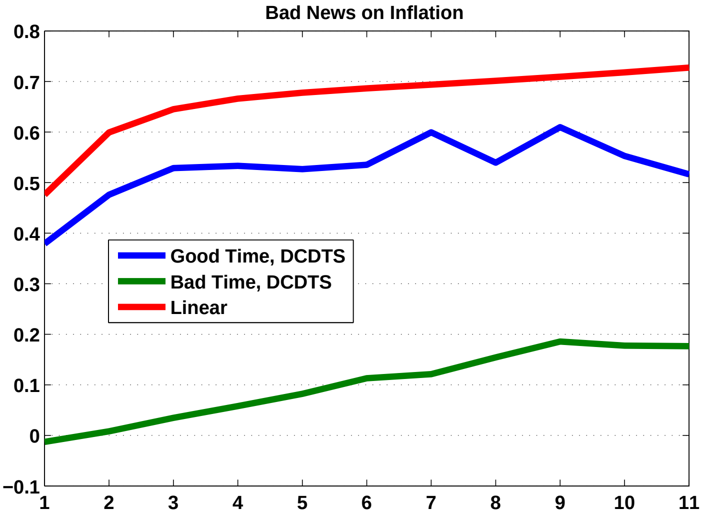
Asymmetric Effects on Yields: Bad news on Employment growth



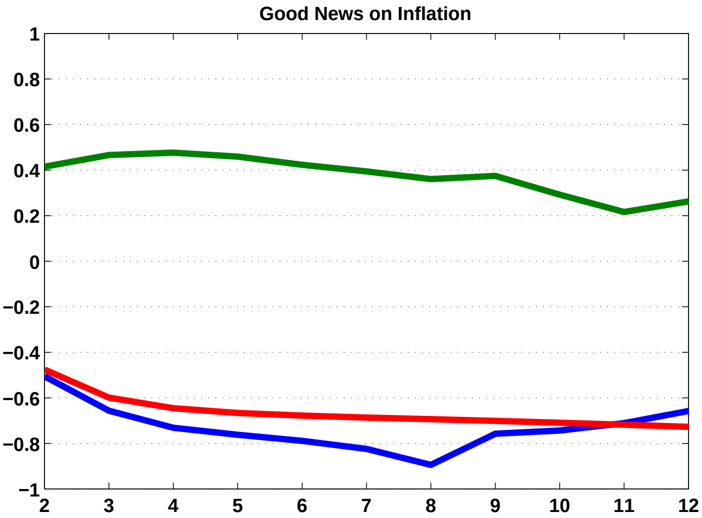
Asymmetric Effects on Yields: Good news on Employment growth



Asymmetric Effects on Yields: Bad news on Inflation



Asymmetric Effects on Yields: Good news on Inflation



Forecasting Yields : the relative RMSE

In sample

Yield Maturity	Forecasting Horizon			
	3	6	9	12
3	0.929	0.946	0.979	1.002
6	0.899	0.943	0.986	1.011
9	0.881	0.939	0.991	1.018
12	0.860	0.932	0.990	1.019

Out-of-Sample

Yield Maturity	Forecasting Horizon			
	3	6	9	12
3	1.003	0.958	1.027	1.065
6	0.980	0.952	1.030	1.072
9	0.912	0.915	1.009	1.060
12	0.837	0.865	0.974	1.039

Conclusion

- We have presented an ordered response model for the Fed fund target rate.
- Our model reflects the reality of the FOMC practice since 1994.
- Contrary to existing ordered response models, our approach allows closed-form term structure of interest rate and derivatives.
- Our model implicitly takes into account well documented non-linearities and asymmetries in the policy and yields responses to shocks in the inflation and economic activity.
- We show that these non-linearities are important for fed-fund rate and yield curve forecasting.

Conclusion

- So far we used a two steps estimation procedure, i.e we did not use term structure of interest rate to estimate the policy response. We are running a single step procedure at this moment.
- In this set-up we did not take into account the impact of fed-fund target rate on economic factors (Inflation and economic activity). We are planning to include r_t in the dynamic of Y_{t+1}.
- Asymmetries are particulary important in pricing interest rate's derivatives. We are now running this application.